Spikes in the Relic Graviton Background from Quintessential Inflation

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Abstract

The energy spectra of gravitational waves (GW) produced in quintessential inflationary models increase in frequency and exhibit a sharp spike around 170 GHz where the associated fraction of critical energy density today stored in relic gravitons is of the order of 10^{-6}. We contrast our findings with the spectra of ordinary inflationary models and we comment about possible detetction strategies of the spike.

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In ordinary inflationary models, $\Omega_{\rm GW}$ (the present fraction of critical energy density stored in relic gravitons) is notoriously quite small. In fact $\Omega_{\rm GW}$ is either flat of decreasing as a function of the present frequency. Therefore, the COBE bound, applied at the frequency scale of the present horizon (i.e. $\nu_0 \sim 1.1 \times 10^{-18}\, h_0$ Hz, $0.5 < h_0 < 1$), demands $h_0^2\, \Omega_{\rm GW} < 6.9 \times 10^{-11}$ [1]. Since the energy spectrum decreases sharply as ν^{-2} between ν_0 and the decoupling frequency, we can further argue that for $\nu > 10^{-16}$ Hz, $h_0^2 \Omega_{\rm GW}$ cannot exceed 10^{-14}. This conclusion can be evaded provided the inflationary phase is not followed immediately by the radiation dominated epoch but rather by an expanding phase driven by an effective source whose equation of state is stiffer than radiation. Indeed a stochastic background of relic gravitons can be produced, with different spectra, in *any* variation of the expansion rate of the Universe [2].

Recently Peebles and Vilenkin discussed a model where the occurrence of a stiff (post-inflationary) phase can be dynamically realized [3]. As previously argued [4] the graviton energy spectra in this class of models must be increasing as a function of the present frequency. One of the motivations of [3] stems from a recent set of observations which seem to imply that $\Omega_{\rm m}$ (the present density parameter in baryonic plus dark matter) should be significantly smaller than one and probably of the order of 0.3. If the Universe is flat, the relation between luminosity and red-shift observed for Type Ia supernovae [5] hints that the missing energy might be stored in a fluid with negative pressure acting as an effective (time dependent) cosmological term whose magnitude should be of the order of 10^{-47} GeV^4, too small if compared with the cosmological constant arising from electroweak spontaneous symmetry bereaking (which would contribute with $(250\ {\rm GeV})^4$). A complementary way of thinking is that the missing energy could come from a dynamical scalar ϕ (the quintessence [6] field) whose potential is unbounded from below [7]. The starting point of [3] is that ϕ could be identified with the inflaton and, as a consequence of this identification the effective potential of ϕ will have to inflate for $\phi < 0$ and it will be unbounded from below for $\phi \geq 0$. As an example we could take

$$V(\phi) = \lambda(\phi^4 + M^4), \quad \text{for } \phi < 0, \quad \text{and} \quad V(\phi) = \frac{\lambda M^8}{\phi^4 + M^4}, \quad \text{for} \quad \phi \geq 0. \tag{1}$$

where, if we want the present energy density in ϕ to be comparable with (but less then) the total (present) energy density we have to require $M \sim 10^6$ GeV. Any other inflationary potential can be used for $\phi < 0$.

For a long period after the end of inflation the kinetic term of ϕ will dominate the stress tensor and, therefore, the effective fluid driving the geometry will have a speed of sound equal to the speed of light. The energy spectra of the relic gravitons will then be blue, namely they increase with frequency with a power wich depends, in general, upon the precise equation of state [4]. Not only gravitons are parametrically amplified in this class of models but also any other (non conformally coupled) scalar degree of freedom [8, 9]. During the stiff phase the energy density of the produced fluctuations red-shifts more slowly than the energy

density of the background, and, at some moment the energy density of the produced quanta will become dominant triggering the reheating of the Universe [8]. GW and inflaton quanta (equaivalent to 3 degrees of freedom) are unable to reheat the Universe on their own: their spectra, are non thermal [4] and cannot thermalize below the Planck scale. If N_s minimally coupled scalar field are present they can reheat the Universe with a thermal distribution since their energy spectra, amplified because of the transition from the inflationary to the stiff phase, can thermalize thanks to non-graviational (i.e. gauge) interactions which get to local thermal equilibrium well below the Planck energy scale. The Universe will become eventually dominated by radiation. This will occur at a temperature which is a function of H_1, the curvature scale at the end of inflation, and of N_s:

$$T_r = \left(\frac{H_1}{M_P}\right) R^{3/4} M_P \simeq 10^3 \; N_s^{3/4} \;\; \mathrm{GeV}, \quad R = N_s R_i, \quad R_i \sim 10^{-2}. \tag{2}$$

If we do not fine-tune H_1 to be much smaller than 10^{-7} in Planck units, T_r is typically a bit larger than 1 TeV. R_i is the fractional contribution of each (minimally coupled) scalar degrees of freedom to the energy density of the produced quanta right after the end of inflation.

In this letter we are interested in the calculation of the energy spectra of the pure (transverse and traceless) tensor modes of the geometry

$$g_{\mu\nu}(\vec{x},\eta) = a^2(\eta)[\eta_{\mu\nu} + h_{\mu\nu}(\vec{x},\eta)], \text{ with } \quad h_{\mu 0} = 0, \quad \nabla_\mu h^\mu_\nu = 0, \quad h^\mu_\mu = 0, \tag{3}$$

where $\eta_{\mu\nu}$ is the usual Minkovski metric and ∇_μ is the covariant derivative associated with the (conformally flat) background geometry. We will focus our attention on the hard branch of the spectrum namely on those tensor modes which left the horizon before the end of inflation and re-entered during the stiff phase. Since GW only couple to the curvature and not to the matter sources (which can only support scalar inhomogeneities) the spectrum will be fully determined by the scale factors whose evolution reads, in conformal time,

$$a_i(\eta) = \left[-\frac{\eta_1}{\eta}\right], \quad \text{for } \eta \leq -\eta_1, \quad \text{and,} \quad a_s(\eta) = \sqrt{\frac{2\eta + 3\eta_1}{\eta_1}}, \quad \text{for} -\eta_1 < \eta \leq \eta_r \tag{4}$$

where $\eta_1 = (a_1 H_1)^{-1}$ and $H_r = (a_r \eta_r)^{-1}$ is the curvature scale at the temperature T_r when the radiation phase commences. Notice that in Eq. (4) the scale factors and their first derivatives (with respect to the conformal time η) are continuous in $-\eta_1$.

The mode function associated with the two polarization of stochastically distributed GW obeys the (Schroedinger-like) equation

$$\psi'' + \left[k^2 - \frac{a''}{a}\right]\psi = 0, \quad \psi = a h, \quad {}' \equiv \frac{\partial}{\partial \eta} \tag{5}$$

which has to be solved in each of the two temporal regions defined by Eq. (4). Given the form of a''/a in the case of Eq. (4), ψ will be a linear combination of Bessel functions, oscillating

for $k^2 \gg |a''/a|$ but parametrically amplified in the opposite limit (i.e. $k < |a''/a|$):

$$\psi_i(k,\eta) = \frac{p}{\sqrt{2k}}\sqrt{x}H_\nu^{(2)}(x), \quad p = \sqrt{\frac{\pi}{2}}e^{-i\frac{\pi}{4}(2\nu+1)}, \quad \eta < -\eta_1,$$
$$\psi_s(k,\eta) = \frac{\sqrt{y}}{\sqrt{2k}}[s^* A_+(k)H_0^{(2)}(y) + sA_-(k)H_0^{(1)}(y)], \quad s = \sqrt{\frac{\pi}{2}}e^{i\frac{\pi}{4}}, \quad -\eta_1 < \eta < \eta_r, \tag{6}$$

where $x = k\eta$ and $y = k(\eta + \frac{3}{2}\eta_1)$; p and s guarantee that the large argument limit of the Hankel functions $H_\nu^{(1,2)}$ is exactly the one required by the quantum mechanical normalization (namely $e^{\pm ik\eta}/\sqrt{k}$). In the case of a pure de Sitter phase $\nu = 1.5$ but corrections (of few percents) can arise if the slow-rolling corrections are taken into account [10].

The graviton energy density per logarithmic interval of frequency will then be given by

$$\rho_\omega = \frac{d\rho_{GW}}{d\ln\omega} = \frac{\omega^4}{\pi^2}\overline{n}(\omega), \qquad \overline{n}(\omega) = |A_-(\omega)|^2, \quad \omega = \frac{k}{a} = 2\pi\nu, \tag{7}$$

where ω is the physical wavenumber and ν the physical frequency. Because of the continuity of $a(\eta)$ and $a'(\eta)$, the two mixing coefficients $A_\pm(k)$ can be fixed by the two conditions obtained matching ψ and ψ' in $\eta = -\eta_1$ with the result that

$$A_-(k) \sim \frac{3\nu}{\pi}2^{\nu-\frac{3}{2}}e^{-\frac{i}{2}\pi(2\nu+1)}\Gamma(\nu)x_1^{-\nu}\ln x_1, \quad \text{valid for} \;\; x_1 < 1. \tag{8}$$

Notice that for $x_1 > 1$ the mixing of the modes is exponentially suppressed and the ultraviolet divergence is avoided [4, 8]. Inserting Eq. (8) into Eq. (7) we get the hard branch of the relic graviton energy spectrum (in critical units)

$$\Omega_{GW}(\omega,\eta_0) = \frac{\rho_\omega}{\rho_c} = \Omega_\gamma(t_0)\ \varepsilon\ \left(\frac{H_1}{M_P}\right)^2\left(\frac{\omega}{\omega_r}\right)\ \ln^2\left(\frac{\omega}{\omega_1}\right), \qquad \omega_r < \omega < \omega_1, \tag{9}$$

which is defined, at the present time η_0, between the two frequencies

$$\nu_r(\eta_0) = 3.58\ R^{\frac{3}{4}}\ \left(\frac{\lambda}{10^{-14}}\right)\left(\frac{g_{\rm dec}}{g_{\rm th}}\right)^{1/3}\text{mHz}, \quad \text{and} \quad \nu_1(\eta_0) = 358\ R^{-\frac{1}{4}}\left(\frac{g_{\rm dec}}{g_{\rm th}}\right)^{1/3}\ \text{GHz}, \tag{10}$$

where

$$\varepsilon = 2R_i\left(\frac{g_{\rm dec}}{g_{\rm th}}\right)^{1/3}, \quad \Omega_\gamma(t_0) = \frac{\rho_\gamma(t_0)}{\rho_c(t_0)} \equiv \frac{g_0\pi^2}{30}\frac{T_0^4}{H_0^2M_P^2} = 2.6\times10^{-5}\ h_0^{-2}. \tag{11}$$

$\Omega_\gamma(t_0)$ is the fraction of critical energy density in the form of radiation at the present observation time; $g_0 = 2$, $T_0 = 2.73$ K; $g_{\rm dec} = 3.36$ and $g_{\rm th} = 106.75$ are, respectively, the number of (massless) spin degrees of freedom at decoupling and at thermalization. The dependence upon the number of relativistic degrees of freedom in $\Omega_{\rm GW}$ occurs since, unlike gravitons, matter thermalizes and then the ratio between the $\rho_{\rm GW}$ and ρ_c is only approximately constant in the radiation dominated phase.

By taking $H_1/M_P = \sqrt{\lambda} \leq 10^{-7}$ the spectrum satisfies the COBE bound [1]. Since the spectral energy density increases sharply in the hard branch the most relevant constraints

will not come from large scales (as in the case of ordinary inflationary models) but from short distance physics and, in particular, from big-bang nucleosynthesis (BBN). In order to prevent the Universe from expanding too fast at BBN we have to demand

$$\int d\ln\omega\,\Omega_{\rm GW}(\omega, t_{\rm n}) < \frac{7}{43}\,(N_{\nu} - 3)\Big[\frac{\rho_{\gamma}(t_{\rm n})}{\rho_{\rm c}(t_{\rm n})}\Big], \tag{12}$$

where $t_{\rm n}$ is the nucleosynthesis time. Since the number of massless neutrinos N_{ν} cannot exceed, in the homogeneous and isotropic BBN scenario is bounded, 3.4 we have that the nucleosynthesis bound implies

$$\frac{3}{N_s}\Big(\frac{g_{\rm n}}{g_{\rm th}}\Big)^{1/3} < 0.07, \tag{13}$$

where the factor of 3 counts the two polarizations of the gravitons but also the quanta associated with the inflaton and $g_{\rm n} = 10.75$ is the number of spin degrees of freedom at $t_{\rm n}$. From Eq. (13), $N_s > 19.9$ as it can occur, for instance, in the minimal supersymmetric standard model (MSSM) where $N_s = 104$ but not in the Minimal Standard model where there is only one Higgs doublet with two (complex) scalars and $N_s = 4$ [3].

From Eqs.(9)–(13), assuming, for instance, $N_s = 21$ the present coordinates of the relic graviton spike, providing an overall normalization of the whole spectrum, will be

$$\nu_1(\eta_0) = 170~{\rm GHz}, \qquad h_0^2\Omega_{\rm GW}(\nu_1, \eta_0) = 0.8\times 10^{-6}, \tag{14}$$

eight order of magnitude larger than the signal provided by ordinary inflationary models [11, 12]. An increase in N_s decreases the height of the spike. The decrease is quite mild since, from Eq. (9) we can deduce that, at the spike, $\Omega(\omega_1, \eta_0) \propto N_s^{-3/4}$. An increase in N_s makes narrower the peak structure associated with the spike. In fact $\nu_r(\eta_0) \propto N_s^{3/4}$ gets larger for larger N_s whereas $\nu_1(\eta_0) \propto N_s^{-1/4}$ gets pushed towards more infra-red values of the spectrum.

In Fig. 1 we report plot the spike computed from Eq. (9) by taking into account the bound of Eq. (13). A decrease in the curvature scale at the end of inflation *does not affect the spike and /or the maximal frequency of the spectrum* since ν_1 does not depend on H_1/M_P. In the case of ordinary inflationary models $\nu_1(\eta_0) = 100\sqrt{H_1/M_P}$ GHz and, for $H_1 \leq 10^{-7}M_P$, $\nu_1(\eta_0)$ can be, at most, 0.1 GHz. In our case, by decreasing H_1, $\nu_1(\eta_0)$ does not change but $\nu_r(\eta_0)$ decreases making the peak broader (in Fig. 1 a decrease in $\nu_r(\eta_0)$ shifts the starting point of the hard branch to the left keeping fixed the position of the spike). For instance if $H_1 = 10^{-7}M_P$ (as assumed in the plot of Fig. 1) the spike is localized according to Eq. (14) and $\nu_r(\eta_0) = 170$ mHz giving a range of fourteen orders of magnitude where the energy density increases as $\omega\ln\omega$.

In the next few years various interferometric detectors like LIGO [13], VIRGO [14], GEO-600 [15] will come in operation. The spectral densities of the the noise are peculiar of each detector but they are all defined between 1 Hz and 1 kHz with a maximal sensitivity around

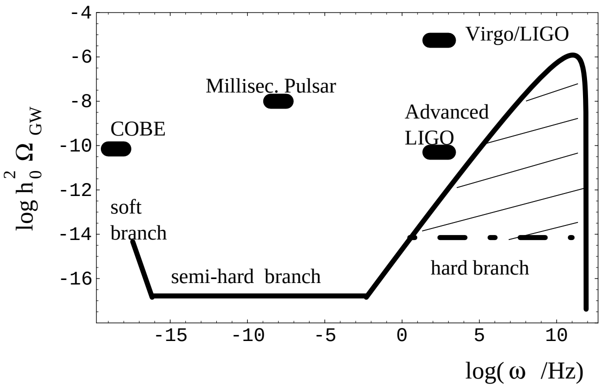

Figure 1: We illustrate the energy spectrum of the relic GW from quintessential inflation as a function of the physical wavenumber. We normalized the height of the spike appearing in the hard branch (see Eq. (9)) to be compatible with the BBN bound. With black spots we indicated the COBE and millisecond pulsar timing bound and the approximate Virgo/LIGO and advanced LIGO sensitivity. The shaded area does corespond to the region where the spike is above the signal provided by ordinary inflationary models. For completeness we indicated also the soft and semi-hard branches of the spectrum whose detailed calculation can be found in [10].

0.1 kHz. In this frequency range the spectral density of the signal, $S_h(\nu)$ can be related to the energy density:

$$\Omega_{GW}(\nu,\eta_0) = \frac{4\pi^2}{3H_0^2}\nu^3 S_h(\nu,\eta_0). \tag{15}$$

Using now Eq. (9) into Eq. (15) we have

$$S_h(\omega,\eta_0) = \mathcal{C}\ R^{-\frac{9}{4}}\ (\frac{g_{\rm dec}}{g_{\rm th}})^{-1}\ \frac{\varepsilon}{\lambda^2}\Omega_\gamma(t_0)\left(\frac{\omega}{\omega_r}\right)^{-2}\ \ln^2\left(\frac{\omega}{\omega_1}\right)\ {\rm Hz}^{-1}, \qquad \omega_r < \omega < \omega_1 \tag{16}$$

with $\mathcal{C} = 6.5\times 10^{-73}\ h_0^2$. For $\omega \sim 0.1$ kHz, $S_h \sim 10^{-52}$–10^{-53} sec. For $\omega \sim 0.01$ kHz, $S_h \sim 10^{-50}$–10^{-51} sec. The spectral density of our signal should be carefully compared with the spectral density of the noise. Our signal is too weak to be interesting for the first generation of interferometers. The sensitivity which is closer to the signal of quintessential inflationary models correposnds to the case of the two upgraded LIGO detectors.

Let us estimate the strength of our background for a frequency of the order of 0.1 kHz –1 kHz. Let us assume that the energy density of the stochastic background is the maximal compatible with the nucleosynthesis indications. As a function of N_s, the GW energy density (in critical units) at a frequency $\nu_I \sim 0.1$–1 kHz is then

$$\Omega_{\rm GW}(\nu_I,\eta_0)\ h_0^2 = 2.29\ 10^{-15}\ N_s^{-3/4}\ [-19.7 + 0.25\ \ln N_s]^2, \qquad \nu_I = 0.1\ {\rm kHz}, \tag{17}$$

$$\Omega_{\rm GW}(\nu_I,\eta_0)\ h_0^2 = 2.29\ 10^{-14}\ N_s^{-3/4}\ [-17.4 + 0.25\ \ln N_s]^2, \qquad \nu_I = 1\ {\rm kHz}. \tag{18}$$

Suppose then that we correlate the two LIGO detectors for a period $\tau = 3$ months. Then, the signal to noise ratio (squared) can be expressed as [16]

$$\left(\frac{S}{N}\right)^2 = \frac{9H_0^4}{50\pi^4}\tau\int_0^\infty d\nu \frac{\gamma^2(\nu)\Omega_{\rm GW}^2(\nu,\eta_0)}{\nu^6 S_{\mathcal{N}}^{(1)}(\nu)S_{\mathcal{N}}^{(2)}(\nu)}, \tag{19}$$

where $\gamma(\nu)$ is the overlap function accounting for the difference in location and orientation of the two detectors. For detectors very close and parallel, $\gamma(\nu) = 1$. For the two LIGO detectors [2] and for other pairs of detectors $\gamma(\nu)$ can be computed [16]. $S_{\mathcal{N}}^{(1,2)}(\nu)$ are the noise spectral densities of LIGO-WA and LIGO-LA and since the two detectors are supposed to be identical we will have that $S_{\mathcal{N}}^{(1)}(\nu) = S_{\mathcal{N}}^{(2)}(\nu)$. In order to detect a stochastic background with 90% confidence we have to demand $S/N \gtrsim 1.65$. For an estimate of S/N we need to evaluate numerically the integral appearing in Eq. (19) where the *theoretical information* comes from $\Omega_{\rm GW}$, given, in our case, by Eq. (9) The *experimental information* is encoded in the noise spectral densities of the LIGO detectors which are not of public availability. We are not aware of any calculation of the sensitivity of the LIGO detectors for an energy spectrum whose frequency behavior is the one of Eq. (9). In the case of a flat energy spectrum the S/N has been computed [17] and we have that the minimum $\Omega_{\rm GW}$ detectable in $\tau = 4$ months is given, with 90 % confidence, by $\Omega_{\rm GW}(\nu_I,\eta_0) = 5\times 10^{-6}h_0^{-2}$ (for the initial LIGO detectors) and by $\Omega_{\rm GW}(\nu_I,\eta_0) = 5\times 10^{-11}h_0^{-2}$ (for the advanced LIGO detectors) [17]. In Fig. 2 we compared our signal given, at the interferometers frequency ν_I, by Eqs. (17) and (18) with the sensitivity of the advannced LIGO project to a flat spectrum. For the allowed range of variation of N_s our signal lies always below (of roughly 1.5 orders of magnitude) the predicted sensitivity for the detection, by the advanced LIGO, of an energy density with flat slope. The main uncertainty in this analysis is however the spectral behavior of the sensitivity for a spectrum which, unlike the one used for comparison, is not flat. It might be quite interesting to perform accuarately the calculation of the S/N in order to see which is the precise sensitivity of the LIGO detectors to a spectral energy density as large as 10^{-12} and rising as $(\nu/\nu_r)\ln(\nu/\nu_1)$ in a frequency range 1 Hz–1 kHz.

On top of the interferometric and resonant bar detectors electromagnetic detectors and, in particular microwave cavities could be employed, in the future, in order to detect background of relic gravitons coming from quintessential inflation. In fact the the nucleosynthesis bound is almost saturated for frequencies of the order of $\nu_1 = 358\times R^{-1/4}$ GHz. Microwave cavities can be used as gravitational waves detectors in the GHz frequency range [18]. There were published results reporting the construction of such a detector [19]. In this prototype $\nu_{\rm GW} = 10$ GHz and the sensitivity to fractional deformations $\delta x/x$ was the order of 10^{-17} using an integration time $\Delta t \sim 10^3$ sec. There, are at the moment, no operating prototypes of these detectors and so it is difficult to evaluate their sensitivity. The example we quoted [19] refers to 1978. We think that possible improvements especially in the quality factors of

[2]One LIGO detector (LIGO-WA) is being built in Hanford (near Washington) the other detector (LIGO-LA) is under construction in Livingston (Lousiana).

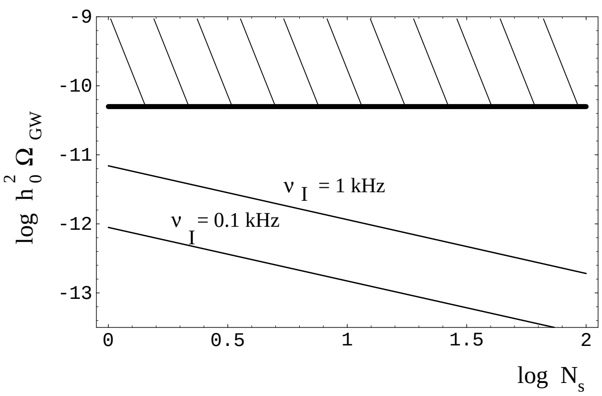

Figure 2: We illustrate the signal of the quitessential graviton background at the frequency of the interferometers. The two thin lines correspond to the signal at $\nu_I = 0.1$ kHz and $\nu_I = 1$ kHz as a function of N_s according to Eqs. (17) and (18). The region between the thin lines represent approximately our signal and the full thick line represents the sensitivity of the advanced LIGO detectors to a stochastic background with flat energy spectrum. In order to be detected our signal should lie in the dashed area. We see that for the allowed range of variation of N_s the signal is always smaller than the sensitivity. This comparison is only illustrative and not completely correct. In fact, the sensitivity our specific energy spectrum (increasing as $\omega \ln \omega$), is not expected to be exactly equal to the sensitivity to a flat $\Omega_{\rm GW}$. The precise sensitivity is, in principle, computable and it requires, according to Eq. (19), the knowledge of the spectral density of the noises which are not publically available.

the resonators can be envisaged. In spite of the fact that improvements can be foreseen we can notice immediately that, perhaps, to look in the highest possible frequency range of our model is not the best thing to do. In fact we can argue that in order to detect a signal of the order of $h_0^2\Omega_{\rm GW} \sim 10^{-6}$ at a frequency of 1 GHz, we would need a spectral density of the noise smaller than the one of the signal, which, from Eq. (15) turns out to be

$$S_h(\nu,\eta_0) \lesssim 9\times 10^{-52}\left(\frac{\rm kHz}{\nu}\right)^3 \;\;{\rm sec}, \tag{20}$$

corresponding to a sensitivity to fractional deformations of the order of 10^{-30}. Moreover, as stressed in [20] and already noticed in [19] the thermal noise is one of the fundamental source of limitation of the sensitivity of these detectors. An interesting strategy could be to decrease the operating frequency range of the device by going at frequencies of the order of 1 MHz.

The common lore is that inflationary models cannot give rise to large energy densities stored in relic gravitons. We showed that this conclusion is in fact evaded if, as in the case of quintessential inflation, a stiff phase follows the inflating epoch. The resulting signal can then be eight orders of magnitude larger than the one obtained in the case of a direct transition from an inflating epoch to a radiation phase. At the LIGO frequency our signal is just below the advanced LIGO sensitivity to flat spectra. Concerning the detectability of our background two final comments are in order. The LIGO-LA/LIGO-WA sensitivity to our specific spectra has not been computed and we wonder if this could be perhaps done in the future. The GHz region (where our signal is maximal) should be carefully explored perhaps with the use of electromagnetic detectors.

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